Math 2D Quiz 6 Afternoon - March 3, 2016

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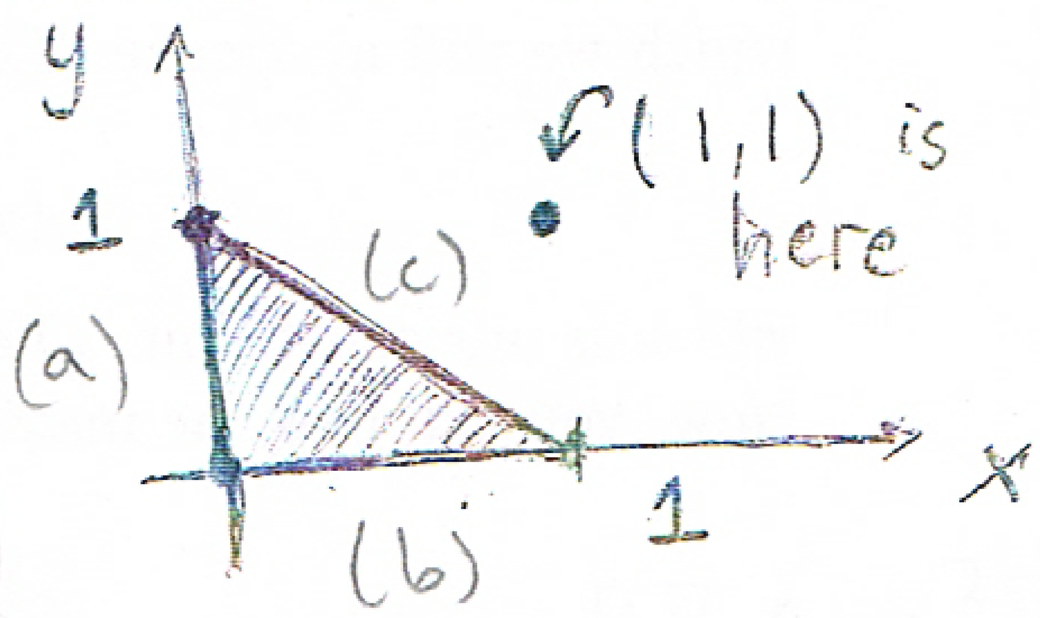
Show all of your work. *There is a question on the back side.

1. Find the absolute maximum and minimum of the function $f(x, y) = x + y - xy$ on the closed triangular region with vertices $(0, 0)$, $(0, 1)$, and $(1, 0)$. You should not use Lagrange Multipliers. It is drawn for you this time. Do not expect regions like this to be drawn for you on the final exam. *Be sure that you go through all of the necessary steps.



1) $\nabla f = 0$: $\nabla f(x,y) = \langle 1-y, 1-x \rangle$

+1 $\nabla f = 0$ at $x=1, y=1$, out of region

2) Boundary: +1 a. $x=0$, $g_a(y) = f(0,y) = y$; $g_a'(y) = 1$ (no local min/max)

b. $y=0$, $g_b(x) = f(x,0) = x$; $g_b'(x) = 1$ (no local max/min)

+1 c. $y = 1-x$, $g_c(x) = f(x,1-x) = \cancel{x} + (1-\cancel{x}) - x(1-x)$

$= 1 - x + x^2$; $g_c'(x) = 2x - 1$, $\boxed{x = \frac{1}{2} \text{ critical pt}}$

$g_c\left(\frac{1}{2}\right) = 1 - \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ so $g_c\left(\frac{1}{2}\right) = f\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{3}{4}$ +1

3) Corners: You can/should use boundary fns

$(0,0) \rightsquigarrow f(0,0) = 0$ (This is also $g_a(0) = 0$, and $g_b(0) = 0$).

$(0,1) \rightsquigarrow f(0,1) = g_a(1) = 1$

$(1,0) \rightsquigarrow f(1,0) = g_b(1) = 1$ +1

$$\boxed{\begin{array}{ll} \text{Abs. Min:} & f(0,0) = 0 \\ \text{Abs. Max:} & 1 \text{ at } f(0,1), f(1,0) \end{array}}$$

2. Repeating the homework: Find the maximum volume of a rectangular box that is inscribed in a sphere of radius R. But, you must use Lagrange Multipliers this time.

Hint: When you solved this in the homework, you had two functions (fill in the blanks):

The first being Volume of the box, $V(x, y, z) =$ xyz

which we will maximize. The second function was a level set $g(x, y, z) = 4R^2$ given by

$$g(x, y, z) = x^2 + y^2 + z^2 = 4R^2$$

which is our constraint. (This g came from the condition that the box is inscribed in the sphere.) Now, you can find the maximum volume with Lagrange Multipliers. Do it.

$\nabla f = \lambda \nabla g$ $\quad \Rightarrow \quad$ $\partial/\partial x$: $yz = 2\lambda x$ (i)

$g = 4R^2$ $\qquad\qquad$ $\partial/\partial y$: $xz = 2\lambda y$ (ii) $\quad$ +2

$\partial/\partial z$: $xy = 2\lambda z$ (iii)

constraint : $x^2 + y^2 + z^2 = 4R^2$ (iv)

Pick on (i): If (i) is $0 = 0$, then on LHS y or $z = 0$

Thus, now assume it's not $0 = 0$. +1 $\Rightarrow$ Volume $= 0$ (min).

☆ For same reason, (ii) and (iii) cannot be $0 = 0$ $\Rightarrow$ Volume $\to 0$.

Hence, we can divide our equs, and we see

$\frac{(i)}{(ii)} \Rightarrow \frac{y}{x} = \frac{x}{y}$; $x^2 = y^2$ $\quad | \quad$ $\frac{(ii)}{(iii)} \Rightarrow \frac{z}{y} = \frac{y}{z}$; $z^2 = y^2$

So, $x^2 = y^2 = z^2$. +1

By (iv), $x^2 + x^2 + x^2 = 4R^2$; $x = \sqrt{\frac{4R^2}{3}} = \frac{2R}{\sqrt{3}}$ and $x = y = z$

Thus, $V(x, y, z)$ is maximal when $x = y = z = \frac{2R}{\sqrt{3}}$

and $\boxed{V_{max} = \frac{8}{3\sqrt{3}} R^3}$ (cube with this side length)

+1